

Solutions to Maths workbook – 1 | Trigonometric Identities and Equations

Level - 2	Daily Tutorial Sheet - 9
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111.(A) As $5 \sin \theta + 3 \sin (\theta - \alpha) = 5 \sin \theta + 3 (\sin \theta \cos \alpha - \cos \theta \sin \alpha) = (5 + 3 \cos \alpha) \sin \theta - 3 \sin \alpha \cos \theta$

$$\text{Now, } -\sqrt{(5 + 3 \cos \alpha)^2 + 9 \sin^2 \alpha} \leq 5 \sin \theta + 3 \sin (\theta - \alpha) \leq \sqrt{(5 + 3 \cos \alpha)^2 + 9 \sin^2 \alpha}$$

$$\Rightarrow \max_{\theta \in R} \{5 \sin \theta + 3 \sin (\theta - \alpha)\} = \sqrt{34 + 30 \cos \alpha}$$

$$\Rightarrow \sqrt{34 + 30 \cos \alpha} = 7 \Rightarrow \cos \alpha = \frac{49 - 34}{30} \Rightarrow \cos \alpha = \frac{1}{2} = \cos \frac{\pi}{3} \Rightarrow \alpha = 2n\pi \pm \frac{\pi}{3}$$

112.(A) Consider : $\sin x (\sin x + \cos x) = n$

$$\Rightarrow \sin^2 x + \sin x \cdot \cos x = n \Rightarrow \frac{1 - \cos 2x}{2} + \frac{\sin 2x}{2} = n \Rightarrow \sin 2x - \cos 2x = 2n - 1$$

$$\text{As } -\sqrt{2} \leq \sin 2x - \cos 2x \leq \sqrt{2} \Rightarrow -\sqrt{2} \leq 2n - 1 \leq \sqrt{2} \Rightarrow \frac{1 - \sqrt{2}}{2} \leq n \leq \frac{1 + \sqrt{2}}{2} \Rightarrow n = 0, 1.$$

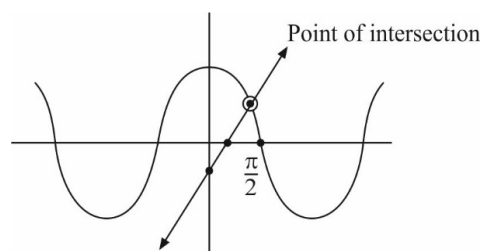
$$\begin{aligned}
 \text{113.(B)} \quad I &= \sum_{r=0}^7 \tan^2 \frac{\pi r}{16} = \tan^2 \frac{\pi}{16} + \tan^2 \frac{2\pi}{16} + \tan^2 \frac{3\pi}{16} + \tan^2 \frac{4\pi}{16} + \tan^2 \frac{5\pi}{16} + \tan^2 \frac{6\pi}{16} + \tan^2 \frac{7\pi}{16} \\
 &= \left(\tan^2 \frac{\pi}{16} + \cot^2 \frac{\pi}{16} \right) + \left(\tan^2 \frac{2\pi}{16} + \cot^2 \frac{2\pi}{16} \right) + \left(\tan^2 \frac{3\pi}{16} + \cot^2 \frac{3\pi}{16} \right) + 1 \quad [\text{As } \tan^2 \frac{4\pi}{16} = 1] \\
 &= \left(\tan \frac{\pi}{16} + \cot \frac{\pi}{16} \right)^2 + \left(\tan \frac{2\pi}{16} + \cot \frac{2\pi}{16} \right)^2 + \left(\tan \frac{3\pi}{16} + \cot \frac{3\pi}{16} \right)^2 - 5 \\
 &= \frac{1}{\sin^2 \frac{\pi}{16} \cdot \cos^2 \frac{\pi}{16}} + \frac{1}{\sin^2 \frac{2\pi}{16} \cdot \cos^2 \frac{2\pi}{16}} + \frac{1}{\sin^2 \frac{3\pi}{16} \cdot \cos^2 \frac{3\pi}{16}} - 5 \\
 &= 4 \left[\frac{1}{\sin^2 \frac{\pi}{8}} + \frac{1}{\sin^2 \frac{4\pi}{16}} + \frac{1}{\sin^2 \frac{6\pi}{16}} \right] - 5 = 4 \left(\frac{1}{\sin^2 \frac{\pi}{8}} + \frac{1}{\cos^2 \frac{\pi}{8}} \right) + 3 = \frac{16}{\sin^2 \frac{\pi}{4}} + 3 = 35
 \end{aligned}$$

114.(A) $\cos x = x - \frac{1}{2}$

Draw graph of both $y = \cos x$ and $y = x - \frac{1}{2}$

At point of intersection values of x and y are same in both

cases so, $\cos x = x - \frac{1}{2}$ when $x \in \left(0, \frac{\pi}{2}\right)$



115.(C) $\frac{x}{\tan(\theta + \alpha)} = \frac{y}{\tan(\theta + \beta)} = \frac{z}{\tan(\theta + \gamma)} = k$

$$\Rightarrow x = k \tan(\theta + \alpha), \quad y = k \tan(\theta + \beta) \text{ and } z = k \tan(\theta + \gamma)$$

$$\therefore \left(\frac{x+y}{x-y} \right) \sin^2(\alpha - \beta) \Rightarrow \left[\frac{\tan(\theta + \alpha) + \tan(\theta + \beta)}{\tan(\theta + \alpha) - \tan(\theta + \beta)} \right] \sin^2(\alpha - \beta)$$

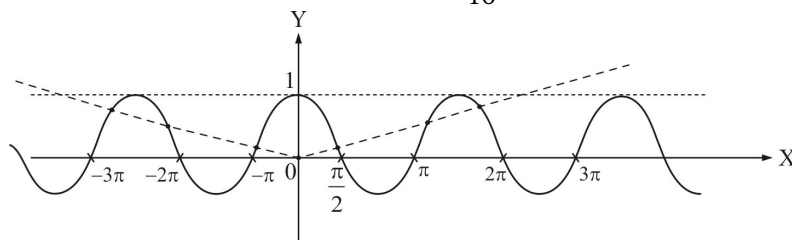
$$\Rightarrow \frac{\sin(2\theta + \alpha + \beta)}{\sin(\alpha - \beta)} \sin^2(\alpha - \beta) \Rightarrow \sin(2\theta + \alpha + \beta) \sin(\alpha - \beta)$$

$$\Rightarrow \sin((\theta + \alpha) + (\alpha + \beta)) \sin((\theta + \alpha) - (\theta + \beta)) \Rightarrow \sin^2(\theta + \alpha) - \sin^2(\theta + \beta)$$

$$\therefore \text{Summation is: } \sin^2(\theta + \alpha) - \sin^2(\theta + \beta) + \sin^2(\theta + \beta) - \sin^2(\theta + \gamma) + \sin^2(\theta + \gamma) - \sin^2(\theta + \alpha) = 0$$

116.(A) Statement 2 is true.

Statement 1: Let $x - 1 = u$ then $\cos u = \frac{|u|}{10}$



Clearly graph of $y = \cos u$ and $y = \frac{|u|}{10}$ cut each other at six points hence total number of solutions is 6.

Hence statement 1 is true.

117.(A) $\tan \alpha + \tan \beta = \frac{1+x}{\sqrt{x}\sqrt{x^2+x+1}}$ and $\tan \alpha \tan \beta = \frac{1}{x^2+x+1}$

$$\Rightarrow \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{\frac{1+x}{\sqrt{x}\sqrt{x^2+x+1}}}{\frac{x^2+x}{x^2+x+1}} = \frac{\sqrt{x^2+x+1}}{(x)^{3/2}} = \sqrt{\frac{1}{x^3} + \frac{1}{x^2} + \frac{1}{x}}$$

118.(B) $\tan[(2n-1)\alpha] = \tan\left(\frac{\pi}{2} - \alpha\right) \quad \because \quad 2n\alpha = \frac{\pi}{2} \quad \therefore \quad \tan \alpha \cot \alpha = 1$

Similarly, product of terms equidistant from beginning & end is 1. Hence, whole equation reduces to 1.

119.(C) $\frac{\cos(\alpha + \beta)}{\cos(\alpha - \beta)} = \frac{\sin(\gamma - \delta)}{\sin(\gamma + \delta)}$ Using componendo and dividendo we get : $\cot \alpha \cot \beta \cot \gamma = \cot \delta$

120.(B) $[(\cos \beta \cos \gamma + \sin \beta \sin \gamma) + (\cos \gamma \cos \alpha + \sin \alpha \sin \gamma) + \cos \alpha \cos \beta + \sin \alpha \sin \beta] = -\frac{3}{2}$

$$1 + 1 + 1 + 2[\sum \cos \alpha \cos \beta + \sum \sin \alpha \sin \beta] = 0$$

$$\cos^2 \alpha + \sin^2 \alpha + \cos^2 \beta + \sin^2 \beta + \cos^2 \gamma + \sin^2 \gamma + 2[\sum \cos \alpha \cos \beta + \sum \sin \alpha \sin \beta] = 0$$

$$(\cos \alpha + \cos \beta + \cos \gamma)^2 + (\sin \alpha + \sin \beta + \sin \gamma)^2 = 0 \Rightarrow \cos \alpha + \cos \beta + \cos \gamma = 0 \quad \text{and} \quad \sin \alpha + \sin \beta + \sin \gamma = 0$$